LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2014**

ST 1503/ST 1501 - PROBABILITY AND RANDOM VARIABLES

Date : 10/11/2014 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

PART – A

Answer **ALL** questions: **(10x2=20 Marks)**

1. Three coins are tossed. Find the probability of getting (i) one head (ii) exactly two heads.
2. If A, B, C are three mutually exclusive and exhaustive events. Find $P(B)$, if $\frac{1}{3}P(C) = \frac{1}{2}P(A) = P(B)$.
3. Define random variable with an example.
4. List the properties of distribution function.
5. State multiplication theorem of probability.
6. Define independent events.
7. Define sample space and events.
8. A continuous random variable X has the p.d.f. f(x) = A $e^{-x/2}$, $x \geq 0$. Find A.
9. What is the mathematical expectation of the sum of the points on 2 dice?
10. Prove that Cov(aX , bY) = ab Cov(X, Y)

PART – B

Answer any **FIVE** questions: **(5x8=40 Marks)**

11. State and prove addition theorem of probability for two events. Extend the result for three events.
12. A bag contains 4 white and 8 black balls. Two balls are drawn at random. What is the probability that (a) both are white (b) both are black (c) one white and one black?
13. Show that E(X + Y) = E(X) + E(Y).
14. Let P(A) = p, P(A | B) = q, P(B | A) = r, find the relations between the numbers p, q and r for the following cases: (a) A and B are mutually exclusive and collectively exhaustive. (b) A is a sub event of B (c) Events A and B are mutually exclusive (d) Events $\bar{A}$ and $\bar{B}$ are mutually exclusive.
15. There are 3 boxes containing 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black balls; 3 white, 1 red and 3 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they come from (i) the first box (ii) second box (iii) the third box

16. The probability function of a random variable X is given by

$$P(X) = \begin{cases} \frac{1}{4} & \text{for } x = -2 \\ \frac{1}{4} & \text{for } x = 0 \\ \frac{1}{2} & \text{for } x = 10 \\ 0 & Otherwise \end{cases}$$

Find the probabilities (a) $P(X \le 0)$ (b) $P(X < 0)$ (c) $P(|X| \le 2)$ (d) $P(0 \le X \le 10)$

17. Find the mean and variance of X whose p.d.f. is $f(x) = 5\, e^{-5x}$, $x \ge 0$.

18. Show that (i) $V(cX) = c^2\, V(X)$, (ii) $V(aX + b) = a^2\, V(X)$.

PART – C

Answer any **TWO** questions: **(2x20=40 Marks)**

19. (a) State and Prove Baye's theorem.

(b) State and prove multiplication law of probability.

20. (a) A continuous random variable X has the p.d.f. $f(x) = \begin{cases} 3x^2 & ,0 < x < 1 \\ 0 & otherwise \end{cases}$

Verify that it is a p.d.f. and evaluate the following probabilities: (i) $P(X \le 1/3)$ (ii) $P(1/3 \le X \le 1/2)$ (iii) $P(X \le ½ \mid 1/3 \le X \le 2/3)$.

(b) Given $F(x) = \begin{cases} 0 & ,x<0 \\ 1 - e^{-x}/4 & ,x \ge 0 \end{cases}$, find

(i) $P(X = 0)$, (ii) $P(X > 0)$, (iii) $P(X > 1)$, (iv) $P(1 < X < 5)$, (v) $P(X = 3)$.

21. (a) Find the mean and variance of a continuous random variable whose p.d.f is

$$f(x) = \begin{cases} 1/8 & ,0 \le x < 2 \\ x/8 & ,2 \le x < 4 \\ 0 & ,otherwise \end{cases}$$

(b) A continuous random variable has the p. d. f. $f(x) = \begin{cases} 6x(1-x) & ,\ 0 \le x \le 1 \\ 0 & ,otherwise \end{cases}$

Determine a number b such that $P(X < b) = P(X > b)$.

22. (a) Explain the following statements: (i) continuous and discrete random variable. (ii) Axioms of probability (iii) Mathematical expectation with suitable example.

(b) State and Prove Chebyshev's inequality.

$$$$$$$